**146.** Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

A random straight line crosses a given ellipse; find the chance that two points, taken at random in the ellipse, shall lie on opposite sides of the line.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $O$ be the center of the given ellipse, axes $OB=a$, $OC=b$. Let $KL$ be the random chord, $GH$ a diameter parallel to $KL$, and $FE$ a diameter conjugate to $GH$ meeting $KL$ in $Q$. Let $OG=n$, $OE=m$, $OQ=z$, $\angle GOE=\phi$, $\angle BOE=\theta$. Equation to ellipse is $x^2/m^2+y^2/n^2=1$.

Area of segment $KEBL=A=2\sin\phi\int ydx$.

$$\therefore A=(2n\sin\phi/m)\int_z^m \sqrt{(m^2-x^2)}dx=\frac{n\sin\phi}{2m}[\pi m^2-2z\sqrt{(m^2-z^2)}-2m^2\sin^{-1}(z/m)]$$

Let $p$ be the chance that both points are on the some side of $KL$, $p'$ the required chance. Also, $\sin\phi=ab/mn$.

$$\therefore\ p=\int_0^m\int_{-m}^m 2A^2dmdz/\pi^2a^2b^2\int_0^m\int_{-m}^m dmdz$$

$$=\frac{1}{4\pi^2m^4}\int_0^m\int_{-m}^m\Big[\pi^2m^4+4z^2(m^2-z^2)+4m^4\Big(\sin^{-1}\frac{z}{m}\Big)^2-4\pi m^2z\sqrt{(m^2-z^2)}$$

$$-4\pi m^4\sin^{-1}\frac{z}{m}+8m^2z\sqrt{(m^2-z^2)}\sin^{-1}\frac{z}{m}\Big]dmdz\div\int_0^m mdm$$

$$=\Big(1-\frac{128}{45\pi^2}\Big)\int_0^m mdm\div\int_0^m mdm=1-\frac{128}{45\pi^2}.$$

$\therefore\ p'=1-p=128/45\pi^2$, the same as for a circle.

**147.** Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

In a bag are $n$ balls, known to be black or white, either color, *a priori*, equally likely. I draw two, which turn out to be one white and one black. I replace them and draw two more. What is the chance both are black?

Solution by W. W. LANDIS, Dickinson College, Carlisle, Pa.

This problem is open to two interpretations. We shall assume that a bag has been chosen at random from $n+1$ bags containing respectively, 0, 1, 2, ....., $n$ black balls. The chance of any particular number of black and white balls being found is then $1/(n+1)$. The chance that the bag containing $r$ black balls is selected and a white and black ball drawn from it then $1/(n+1).2.r/n.(n-r)/(n-1)$. Adding these probabilities from $r=1$ to $r=n-1$ we obtain as sum $\frac{1}{3}$. As this event has happened, the probabilities in the next case must be multiplied by 3. We get then for the probability of drawing two black balls from this bag

$$\frac{6}{n+1}\cdot\frac{r}{n}\cdot\frac{n-r}{n-1}\cdot\frac{r}{n}\cdot\frac{r-1}{n-1}=6[(n+1)r^3-r^4-nr^2]/(n+1)n^2(n-1)^2.$$